A modified vacation queueing model and its application on the Discontinuous Reception power saving mechanism in unreliable Long Term Evolution networks

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ABSTRACT

In this paper, Discontinuous Reception, a power saving mechanism in Long Term Evolution wireless networks, is analytically investigated in the presence of transient faults by using an unreliable $M/G/1$ queue with multiple vacations. System failures may occur either due to transient faults in the mobile device or due to wireless link errors. When a failure occurs, the device may request the retransmission of the packet after a recovery period. In such a case, the power consumption will be further increased. Steady state analysis is presented, and decomposition results are discussed. Energy and performance metrics are obtained, and used to provide useful numerical results.

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1. Introduction

Wireless networks have enjoyed an exponential growth and wireless communication has become an essential part of modern life. Many new wireless applications demand higher data rates and consume more energy than in the recent past. However, wireless devices are always powered by batteries, which have limited lifetime and constrain their use and the growth of wireless networks.

Recent advances in wireless communications are marked by the development of Fourth Generation (4G) technologies such as 3GPP LTE (3rd Generation Partnership Project Long Term Evolution) [1]. As LTE increases data rates by a factor of 50 compared to 3G, wireless device batteries are still about the same size, so substantial improvements in power usage are necessary to operate at these very high rates and wide frequency bands. These savings come from improvements in the hardware, the system architecture, and the protocols used. Consequently, power saving mechanisms are becoming increasingly important for next-generation wireless networks. In order to improve client's operation period without recharging the battery of the device, 3GPP LTE defines the Discontinuous Reception (DRX) mode [1].

In DRX mode, the User Equipment (UE, the mobile device) powers down most of its circuitry for a predefined period (DRX cycle) when there are no packets to be received. A DRX cycle consists of a sleep period, during which the UE is unavailable, and an on-period (or a listening period), during which the UE monitors new packet arrivals in the evolved Node B (eNB, the base station) by receiving an indication message conveyed via a control channel. If the UE receives a positive indication message implying that newly arrived packets are buffered and ready for transmissions, it wakes up to receive the buffered packets. Otherwise, it immediately goes back to sleep to prevent redundant power consumption.

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Wireless/mobile systems are notorious for the highly time-varying and unreliable wireless channels due to the propagation effects such as multipath fading, shadowing, and path loss. Moreover, mobile devices often suffer from transient faults such as memory or software faults. Faults are generally classified either as transient or as permanent (see [2]). A transient fault will eventually disappear without any apparent intervention (a restart of the device is usually enough in case of a software fault, whereas a failed wireless link becomes again operational soon), whereas a permanent one remains until it is removed by some external agency. In this work we consider only transient faults that recur unpredictably. Transient faults cause system failures and a traditional way to increase reliability is repetition.

However, retransmission schemes in modern wireless networks employ forward error correction (FEC), a technique used for controlling errors in data transmission. Under such schemes, the total number of retransmissions (until successful transmission) can be reduced substantially. Clearly the repetition of the transmission process will increase the power consumption and thus, a cross-layer investigation of the effect of failures on power consumption is critical.

1.1. Related work in energy saving in wireless systems

Several analytical studies have been conducted in the literature regarding the performance of DRX. Yang et al. [3] provided a Markov chain model to investigate the performance of the DRX in UMTS (which is the predecessor technology of 3GPP LTE) systems. There, the DRX mechanism was modeled as an $M/G/1$ queue with multiple vacations. In [3] the authors derived the Laplace transform of the downlink packet delay and the power saving factor. Yang et al. [4] proposed a semi-Markov model where packet arrivals followed the European Telecommunications Standards Institute (ETSI) bursty packet traffic model, while more recently Baek and Choi [5] studied a discrete time model to analyze DRX in 3GPP LTE by taking into account both the downlink and the uplink traffic. Mancuso and Alouf [6] analyzed a queueing model for the behavior of users and base stations in continuous connectivity, while Anisimov et al. [7] provided a comparison between the DRX in LTE-Advanced and the sleep mode that is employed in the IEEE 802.16m standard.

In order to maximize energy efficiency, a traffic-based DRX cycle adjustment scheme was proposed by Yu and Feng [8] to adjust the DRX parameters according to traffic estimation. A partially observable Markov decision process was employed to conjecture the traffic status and to provide the selection policy for DRX parameters. Zhou et al. [9] modeled an adjustable DRX power saving mechanism under bursty packet data traffic by using a semi-Markov process. Their analytical results, which were validated against simulations, showed that DRX in LTE achieves better power saving gains over DRX in UMTS, at the price of a prolonging wake up delay. Finally, Yeh et al. [10] provided a comparative analysis of energy saving methods in 3GPP and 3GPP2 standards.

Power saving modes in systems other than the LTE have also been studied; hereafter we cite some of these studies. A queueing-based modeling framework was used for the analysis of power saving of IEEE 802.16e in [11–14]. The power saving operation of the Cellular Digital Packet Data (CDPD) system was studied using queueing theory in [15,16]. Sleep mode operation of the IEEE 802.11 protocol was investigated in [17], while the energy efficiency in Wireless Sensor Networks (WSN) using vacation models was studied in [18–20].

1.2. Related work in energy saving in wired systems

Clearly, much study has been devoted to power saving in wireless networks where the nodes are battery operated. As indicated in [21], the radio transceiver, which is the dominant source of energy consumption within a node, consumes almost the same amount of energy when it exchanges packets and when it is in the idle mode. Therefore, switching off the transceiver can result in meaningful energy savings. Furthermore, the authors in [22] introduced a novel energy efficient algorithm to find and maintain routes in mobile ad-hoc networks. This work was motivated by learning algorithms and the concept of smart packets from previous research on cognitive packet networks [23].

In contrast to wireless networks, wired networks, which are massive consumers of power have recently drawn attention. The authors in [24] examined some approaches, which were used for dynamically managing wired packet networks, in order to minimize energy consumption, while meeting users' quality of service (QoS) needs. Their approach was based on automatically turning on/off link drivers and/or routers in response to changes in network load.

In [25,26], analytical models based on G-networks [27], were developed to incorporate the effects of user traffic, the overhead in QoS and the energy consumption introduced by the control traffic, that will be needed to carry out the rerouting decisions. Building on these works, they also examined in [28] the use of a gradient-based algorithm for QoS and power minimization in wired networks. Two distinct schemes, the conventional shortest-path routing and an autonomic energy aware routing algorithm [29], were investigated as the starting point for the gradient algorithm.

Recently, Gelenbe [30,31] proposed the Energy Packet Network (EPN), which stores and forwards quantized energy units to and from a large range of devices. The theoretical framework was based on G-networks. Finally, Gelenbe and Lent [32] studied ways to reduce energy consumption in information and communication technology systems while preserving acceptable levels of QoS.

1.3. Our contribution

In this work, we investigate the performance of the DRX power saving mode in LTE systems under the presence of transient faults such as transient software faults of the device and wireless link errors. Clearly none of the studies presented

above proposed an analytical model that incorporates the critical and realistic concept of unreliability in power-aware mobile wireless systems.

We propose and analyze in detail an unreliable $M/G/1$ queue with multiple vacations, a timer, start-up and close-down times. The multiple vacation scheme captures the operation of short and long DRX cycles, while the service process properly describes the operation of the Hybrid Automatic Repeat reQuest (HARQ) scheme which is a combination of the ARQ and the FEC algorithm. Under this scheme, if the errors in the received packet can be fixed by the FEC, no retransmission is requested. In our model we employ this feature, and upon a failure (due to transient faults), if the errors can be fixed, no retransmission is required. On the other hand, if the erroneous packet cannot be fixed, a retransmission is requested. Using energy metrics, we provide extensive numerical results that give an insight into the operation and the performance of the system.

We also provide an alternative method to compute useful performance measures such as the expected waiting time (and as a result the expected number of packets in the system) using the powerful Mean Value Analysis (MVA) method. Using this methodology we can avoid the use of the generating function technique. Following the authors in [33], we finally prove that our model satisfies the stochastic decomposition property.

The most related paper on that subject is the one by Yang et al. [3]. In contrast to our model, Yang et al. [3] studied the performance of DRX in UMTS systems (which consists of a single DRX period) without taking into account the realistic concept of transient faults, which are common in wireless systems. Moreover, they also assumed that both the time periods to receive the approval from the base station to enter the sleep mode (note that, optionally, the eNB may send a DRX Command MAC control element to the UE to initiate the DRX mode [34]) and to wake up are negligible.

To the author's best knowledge, it is the first time that the DRX power saving method in LTE systems is studied using such a sophisticated queueing model. More precisely, all the related papers in that field do not take into account the time that is necessary for the device to receive the approval (through a sleep response message) from the base station to enter sleep mode and to turn off most of its physical components (close-down), as well as the time that is necessary for the device to turn on its components in order to return to awake mode (start-up) [34,35]. Note that the close-down period has also an impact on QoS, because it can be thought of as an extended inactivity period. Thus, if a packet arrives during that period, it will be transmitted to the UE at a cost of a delay due to the start-up process.

In contrast to other papers related to the performance of DRX in LTE systems, we take into account for the first time the realistic concept of transient faults and the close-down, start-up periods. In our model, in the presence of failures, if the erroneous packet can be fixed by the FEC algorithm, the device does not request a retransmission. If the packet errors cannot be fixed, a retransmission is required. The form of the service process captures the operation of retransmissions and especially the chase combining ([36], pp. 120–125) HARQ scheme, where the same coded packet is retransmitted.

The rest of the paper is organized as follows. In Section 2 we briefly describe the DRX power saving mode in LTE systems. A detailed description of the model under consideration is given in Section 3, and some very important preliminary results are obtained in Section 4. The embedded Markov chain model, the waiting time analysis and some decomposition results are investigated in Section 5. In Section 6, interesting performance and energy metrics are obtained in closed-form and an alternative method to obtain performance measures using the mean value analysis is proposed. Finally, in Section 7 we provide extensive numerical results that give an insight into the operation of the system.

2. Basic operation of DRX in 3GPP LTE

A simplified LTE system [37] consists of the evolved Packet Core (ePC) and the evolved UMTS Terrestrial Radio Access Network (eUTRAN). The ePC is responsible for the overall control of the UE, while the eUTRAN handles the radio connectivity and its main component is the eNB. In the eUTRAN, DRX is managed by the Radio Resource Control (RRC) which has two state modes, the RRC-Idle and the RRC-Connected. This work focuses on the DRX in RRC-Connected mode consisting of alternating DRX mode and awake mode. In the RRC-Connected mode, the UE is registered with its serving eNB and returns from the DRX to the awake mode without a reentry process.

In the basic operation of DRX in LTE, upon expiration of the inactivity timer, the UE enters the short DRX period (the sleep mode). The short DRX period consists of at most N short DRX cycles. Each cycle consists of a sleep period and an on-period which alternate. Upon expiration of the inactivity timer, the UE enters a sleep period, during which it remains powered down and thus cannot be reached by the eNB. At the end of this period, the UE is briefly reactivated (on-period: an interval of fixed length) to check (be informed by the Physical Downlink Control Channel (PDCCH)) whether there are packets waiting in the eNB's buffer. If not, the UE enters a second sleep period, which has the same length as the first one and so on. However, if any packets have arrived at the eNB's buffer during the last sleep period, the UE wakes up and the eNB transmits all the packets exhaustively. After that, the whole procedure is repeated.

If the number of consecutive short DRX cycles reaches a fixed number N, the UE enters the long DRX period, which consists of at least one long DRX cycle. The operation of the long DRX period (cycle) is similar to the short DRX period (cycle). The only difference is that the length of the sleep period in a long DRX cycle is larger compared with the short DRX cycle, whereas the on-period has the same length as in the short DRX cycle. As a result, the UE can save more energy during the long DRX period (cycle).

It must be noted that during the sleep period (either of a short or of a long DRX cycle), the eNB will not transmit any packets, since, the UE is powered down during that period.
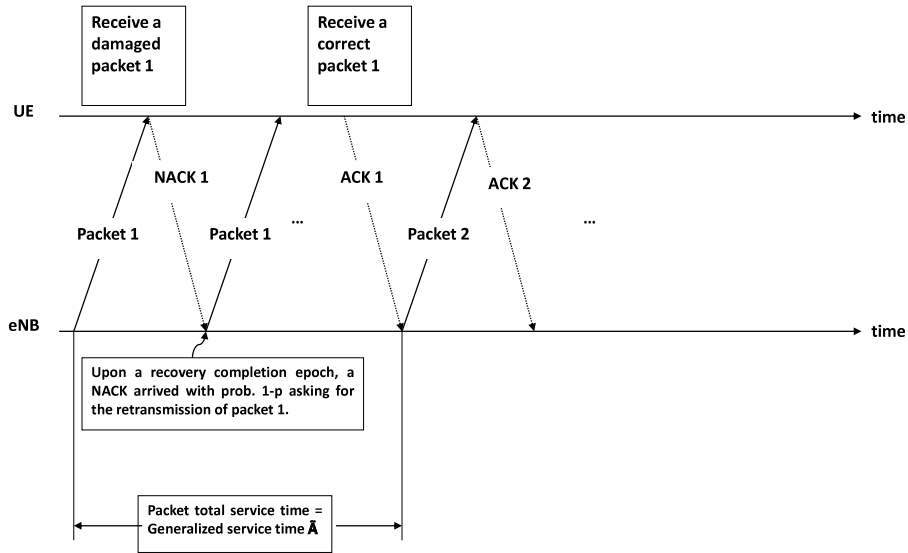
Fig. 1. Timing diagram for HARQ.

3. The mathematical model

We assume that packet arrivals at the eNB's buffer form a Poisson process with rate $\lambda > 0$. Let B be the "service time" of a packet, i.e., the time which is necessary for a packet to be delivered to the UE. Therefore, we consider the eNB as a First In–First Out (FIFO) server. Service time B is arbitrarily distributed with cumulative distribution function (cdf) $B(x)$, probability density function (pdf) $b(x)$, Laplace–Stieltjes Transform (LST) $\beta^*(s)$ and moments $\bar{b}, \bar{b}^{(2)}$.

A system failure is caused due to one or more faults, which are assumed to be transient and are governed by a Poisson process with rate $\theta > 0$. As transient faults, we mean faults such as memory/software faults of the UE or wireless link errors [2,38]. The nature of these faults is such that they will eventually disappear without any apparent intervention. For simplicity, we assume that a UE failure occurs once a fault has occurred.

The transmission procedure in LTE systems is based on the HARQ algorithm, which is a combination of the ARQ and the FEC technique. In such a case, a retransmission is requested only if the packet errors cannot be corrected by the FEC. There are several ways of combining retransmission and channel error coding. In our case, we adopt the chase combining method ([36], pp. 120–125), where the same coded packet is retransmitted.

At a failure epoch, the service stops and the UE needs a time period R to recover (e.g. a possible restart). R is arbitrarily distributed with cdf $R(x)$, pdf $r(x)$, LST $r^*(s)$ and moments $\bar{r}, \bar{r}^{(2)}$. Thus, the service time of a packet will be delayed due to the occurrence of transient faults (see Fig. 1). The FEC can fix the erroneous packet, and no retransmission is required with probability p (the UE sends a positive acknowledgment (ACK) to the eNB), whereas the erroneous packet cannot be fixed and the retransmission will be initiated at once with probability $1 - p$ (the UE sends a negative acknowledgment (NACK) to the eNB and asks for the retransmission of the packet).[1]

When all the packets in the buffer are transmitted successfully, the UE enters the inactivity period I, by enabling a timer. I is arbitrarily distributed with cdf $I(x)$, pdf $i(x)$, LST $i^*(s)$ and moments $\bar{i}, \bar{i}^{(2)}$. If a packet arrives at the eNB during I, the timer stops and the eNB transmits the packet (see Fig. 2). If no packets arrive during I, the UE enters a close-down period, during which it sends a sleep request message to the eNB.

During the close-down period, the eNB receives the sleep request from the UE and sets the DRX parameters. Note that, optionally, the eNB may send a DRX Command MAC control element (an approval through a sleep response message) to the UE to initiate the DRX mode [34].

Therefore, the close-down period comprises the time period for the UE to send a sleep request to its serving eNB, plus the time period to receive the approval through a sleep response message from the eNB plus the time to turn off most of its physical components. The duration of the close-down period C is arbitrarily distributed with cdf $C(x)$, pdf $c(x)$, LST $c^*(s)$ and moments $\bar{c}, \bar{c}^{(2)}$. If a packet arrives at the eNB during the close-down period, the eNB does not allow the UE to enter the sleep mode and transmits the packet (see Fig. 3). A start-up is necessary for gaining the access right for the link. The start-up period S is arbitrarily distributed with cdf $S(x)$, pdf $s(x)$, LST $s^*(s)$ and moments $\bar{s}, \bar{s}^{(2)}$.

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[1] Note that this situation is equivalent to the case where two types of faults may occur during the reception of a packet. The first type faults can be fixed by the FEC and they arrive at rate θp, while the second type faults arrive at rate $\theta(1 - p)$ and cannot be corrected.
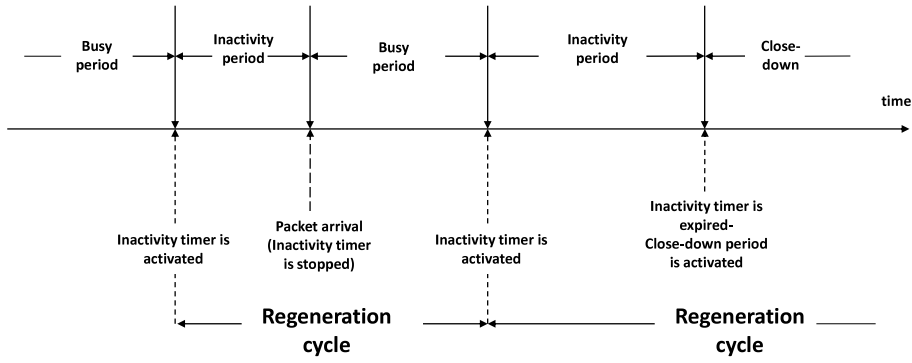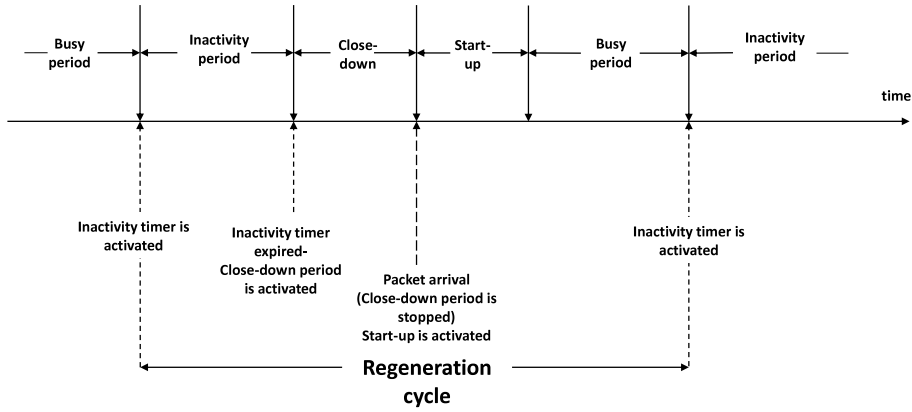
Fig. 2. Timing diagram I.
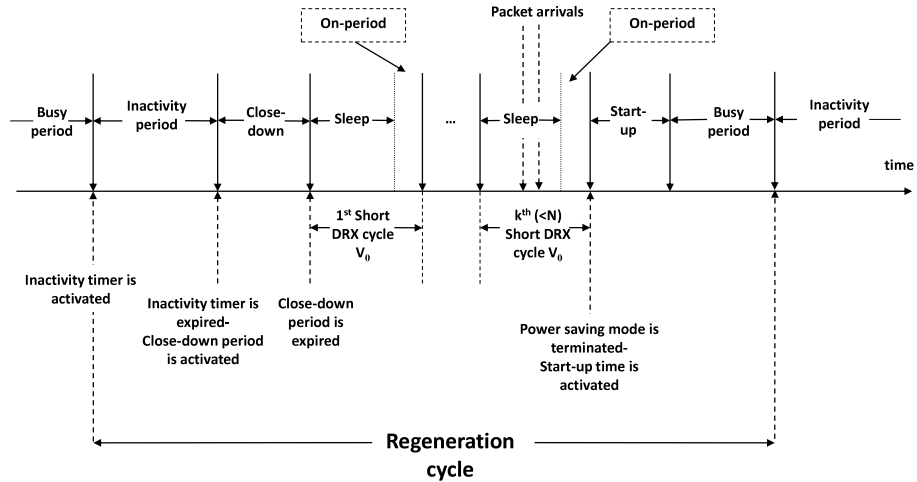

Fig. 3. Timing diagram II.

Fig. 4. Timing diagram III.

Upon expiration of the close-down period, the UE enters the short DRX period, which consists of at most N short DRX cycles. Each short DRX cycle consists of a sleep period and an on-period, which alternate. The duration of a short DRX cycle V_0 is arbitrarily distributed with cdf $V_0(x)$, pdf $v_0(x)$, LST $v_0^*(s)$ and moments $\overline{v}_0$, $\overline{v}_0^{(2)}$. At the end of the sleep period, the UE wakes up to listen to the PDCCH for a time period of fixed length τ. If packets have arrived during the last sleep period, the UE starts to receive them after the expiration of a required start-up period (see Fig. 4). Otherwise, the UE will enter the next short DRX cycle.

If the number of consecutive short DRX cycles reaches a fixed number N, the UE enters the long DRX period, which consists of at least one long DRX cycle of duration V_1. V_1 is arbitrarily distributed with cdf $V_1(x)$, pdf $v_1(x)$, LST $v_1^*(s)$ and
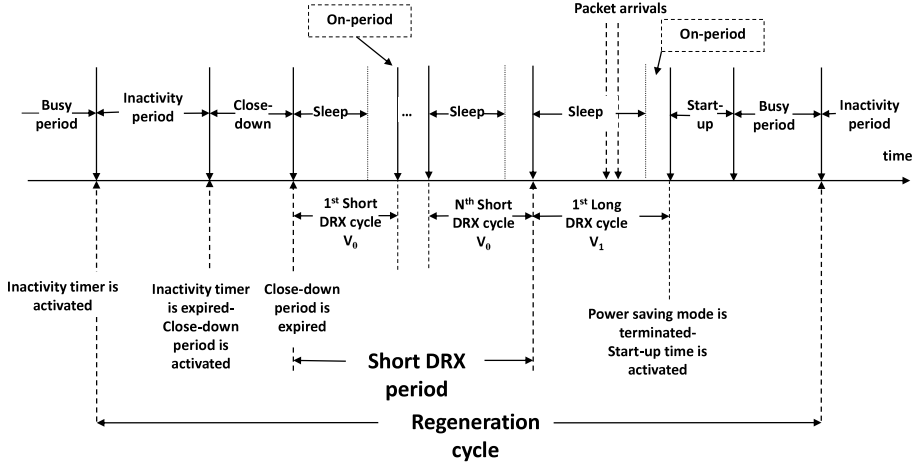
Fig. 5. Timing diagram IV.

moments $\overline{v}_1$, $\overline{v}_1^{(2)}$. The duration of the long DRX cycle is larger compared with the short DRX cycle and consists of a sleep period and an on-period. We assume that the on-period of a long DRX cycle equals to the one of a short DRX cycle (τ). The operation of the long DRX period is similar to the short DRX period (see Fig. 5 where the short DRX period has expired and the UE enters the long DRX period. During the first long DRX cycle some packets have arrived and the sleep mode is terminated).

Before proceeding with the analysis, we have to make the following remarks:

- In this paper, the concept of close-down period refers to the time that is necessary for the device to send a sleep request to the eNB, plus the time to receive the approval from the eNB to enter a sleep period plus the time to turn off most of its physical components. However, we may consider the close-down period as the time for the UE to transmit a packet or to generate a call (uplink-outgoing call). This periodical call may be interrupted due to the arrival of calls (downlink packets) in the eNB. The interested reader is referred to a recent paper by Artalejo and Phung-Duc [39] that deals with the analysis of such systems (two-way communication).
- Consider the case where we neglect the principles of close-down and start-up periods. Assume also that $V_0 \backsim V_1 \backsim V$ (that is, the short and the long DRX cycles are independent and identically distributed random variables) and that the inactivity period cannot be preempted by packet arrivals. In such a case, our model becomes a special case of a queueing model with a server of walking type, introduced in [40]. More precisely, when the server finishes the service of the last packet in the queue, it "takes a walk" (the inactivity period), after which it returns to inspect the queue. If the queue is not empty, the first packet in line is served. Otherwise, the server goes for a vacation (V). In the latter case, when the vacation period expires, the server inspects the queue, and it begins a service cycle in case the queue is not empty. If the queue is empty, the server goes again for a vacation V.

4. General results

In this section, we derive some useful results that are essential for the analysis that follows.

Let us define the generalized service time $\tilde{A}$ as the elapsed time from the epoch the eNB starts the delivery of a packet, until the epoch it is ready for a "new service". The "new service" refers to the delivery procedure of a new packet. Therefore, that period will be terminated with the successful delivery of a packet. Denote also as $N(\tilde{A})$ the number of packets that arrive during $\tilde{A}$, and $a_j(t)dt = P(t < \tilde{A} \leq t + dt, N(\tilde{A}) = j)$.

If $y_k(t) = e^{-\lambda t} \frac{(\lambda t)^k}{k!}$, then

$$a_j(t) = e^{-\theta t} y_j(t) b(t) + \theta e^{-\theta t} p \sum_{k=0}^{j} y_k(t)(1 - B(t)) * y_{j-k}(t) r(t)$$

$$+ \theta e^{-\theta t}(1 - p) \sum_{k=0}^{j} y_k(t)(1 - B(t)) * \sum_{m=0}^{j-k} y_m(t) r(t) * a_{j-k-m}(t), \tag{1}$$

where "$*$" means convolution.

Let us explain the right hand side of (1). The first term refers to the case where no failure occurred during the service of a packet. The second term refers to the case where a failure occurred, but the erroneous packet was fixed by the FEC algorithm and no retransmission was requested, upon the completion of the recovery period. The third term corresponds to the case

where a failure occurred during the service, but the packet errors were not fixed by the FEC. As a result, a retransmission was requested upon the recovery of the system (see also Fig. 1, which describes the operation of HARQ algorithm).

Define further $a^*(z, s) = \int_0^\infty e^{-st} \sum_{j=0}^\infty a_j(t)z^j dt$. Therefore, after manipulations

$$a^*(z, s) = \frac{\beta^*(s + \theta + \lambda - \lambda z) + \theta p \frac{1 - \beta^*(s+\theta+\lambda-\lambda z)}{s+\theta+\lambda-\lambda z} r^*(s + \lambda - \lambda z)}{1 - \theta(1 - p)\frac{1 - \beta^*(s+\theta+\lambda-\lambda z)}{s+\theta+\lambda-\lambda z} r^*(s + \lambda - \lambda z)}. \tag{2}$$

Let us also define

$$\rho = \left.\frac{\partial a^*(z, 0)}{\partial z}\right|_{z=1} = \frac{\lambda(1 - \beta^*(\theta))(1 + \theta\overline{r})}{\theta\left(\beta^*(\theta) + p(1 - \beta^*(\theta))\right)}. \tag{3}$$

The function $a^*(z, 0)$ arises in the Markov chain analysis which follows (Section 5). There, in order to proceed with the analysis we will have to investigate the number and the location of the roots of $z - a^*(z, 0) = 0$ inside the unit disk. The following theorem addresses this issue, and its proof is based on a well-known result given by Takacs [41]. In order to prove this theorem, we have to show first that $a^*(z, s)$ is an analytic function in its domain. Note that this is ensured by Eq. (2). Then,

Theorem 1. *If* (i) $\Re(s) > 0$, (ii) $\Re(s) \geq 0$, *and* $\rho > 1$ *the equation*

$$z - a^*(z, s) = 0, \tag{4}$$

has one and only one root, say $z = x(s)$, *inside the region* $|z| < 1$. *Specifically for* $s = 0$, $x(0)$ *is the smallest positive real root of* (4) *with* $x(0) < 1$ *if* $\rho > 1$ *and* $x(0) = 1$ *for* $\rho \leq 1$.

Proof. Clearly, $a^*(z, s)$ is the LST of a probability generating function (pgf), and thus an analytic function in its domain. Then, for the closed contour $|z_2| = 1$ and under the assumption (i) we have always

$$\left|a^*(z, s)\right| \leq a^*(z, \Re(s)) < a^*(1, 0) = 1 \equiv |z|,$$

while for $\Re(s) \geq 0$, we need to consider the closed contour $|z| = 1 - \epsilon$ ($\epsilon > 0$ a small number). In such a case,

$$\left|a^*(z, s)\right| \leq a^*(1 - \epsilon, \Re(s)) < 1 - \epsilon \equiv |z|, \tag{5}$$

only if in addition

$$\frac{d}{d\epsilon}a^*(1 - \epsilon, 0)\,|_{\epsilon=0} = -\rho < \frac{d}{d\epsilon}(1 - \epsilon)\,|_{\epsilon=0} = -1,$$

or we need now $\rho > 1$ for the relation (5) to hold. A final reference to Rouché's theorem completes the first part of the proof.

Moreover, for $s = 0$ the convex function $a^*(z, 0)$ is a monotonically increasing function of z, for $0 \leq z \leq 1$, taking the values $a^*(0, 0) < 1$ and $a^*(1, 0) = 1$. Thus, $0 < x(0) < 1$ if $\rho > 1$, whereas for $\rho \leq 1$, $x(0)$ becomes equal to 1 and this completes the proof. $\square$

In the sequel we will provide some results that are necessary for the determination of the pdf of the regeneration cycle. The regeneration cycle, say $\widetilde{W}$, is the elapsed time from the epoch an inactivity period is enabled, until the epoch the next inactivity period is about to begin.

Let $\widehat{V}$ be the time interval from the epoch the UE enters the short DRX period until the epoch the eNB is ready to deliver the first packet to the UE. Define also $N(\widehat{V})$ to be the number of packets that arrive during $\widehat{V}$ and $\widehat{v}_j(t)dt = P(t < \widehat{V} \leq t + dt, N(\widehat{V}) = j)$. Then,

$$\widehat{v}_j(t) = \sum_{k=1}^j y_k(t)v_0(t) * y_{j-k}(t)s(t) + e^{-\lambda t}v_0(t) * \sum_{k=1}^j y_k(t)v_0(t) * y_{j-k}(t)s(t) + \cdots$$

$$+ \underbrace{e^{-\lambda t}v_0(t) * \cdots * e^{-\lambda t}v_0(t)}_{N-1} * \sum_{k=1}^j y_k(t)v_0(t) * y_{j-k}(t)s(t)$$

$$+ \underbrace{e^{-\lambda t}v_0(t) * \cdots * e^{-\lambda t}v_0(t)}_{N} * \left[\sum_{k=1}^j y_k(t)v_0(t) * y_{j-k}(t)s(t)\right.$$

$$\left. + e^{-\lambda t}v_1(t) * \sum_{k=1}^j y_k(t)v_1(t) * y_{j-k}(t)s(t) + \cdots\right]. \tag{6}$$

Let us explain the right hand side of (6). The first term refers to the case which at least one packet has arrived during the first short DRX cycle V_0. In such a case, upon the termination of V_0, a start-up is enabled. In the second term, the first short

DRX cycle has expired without packet arrivals, and upon its completion, the UE enters the second DRX cycle during which k packets ($1 \leq k \leq j$) have arrived etc.

The third term corresponds to the case where, $N - 1$ consecutive short DRX cycles have expired without packet arrivals, and during the Nth DRX cycle, at least one packet has arrived. Upon the completion of the corresponding DRX cycle, the UE enables the start-up in order to receive the packets. Finally, in the fifth term, N consecutive short DRX cycles have expired without packet arrivals, and the short DRX period is terminated. Then, the UE enters the long DRX period that consists of at least one long DRX cycle (the terms inside the square brackets).

Therefore, if $\widehat{v}^*(z, s) = \sum_{j=1}^{\infty} \int_0^{\infty} e^{-st} \widehat{v}_j(t) z^j dt$ then,

$$\widehat{v}^*(z, s) = s^*(s + \lambda - \lambda z) \left\{ \frac{\left(1 - (v_0^*(s + \lambda))^N\right)\left(v_0^*(s + \lambda - \lambda z) - v_0^*(s + \lambda)\right)}{1 - v_0^*(s + \lambda)} \right.$$

$$\left. + \frac{\left(v_0^*(s + \lambda)\right)^N \left(v_1^*(s + \lambda - \lambda z) - v_1^*(s + \lambda)\right)}{1 - v_1^*(s + \lambda)} \right\}. \tag{7}$$

Let the busy period be the time period from the epoch a service is initiated, until the epoch the server is ready to enable the inactivity timer. Let $\mathbf{H}^{(i)}$ be the duration of the busy period of the server, initiated by i packets, and $g^{(i)}(t)dt = P(t < \mathbf{H}^{(i)} \leq t + dt)$. Then, following the lines in Takacs [41] (pp. 60–63), we obtain

$$g^{(i)}(s) = \int_0^{\infty} e^{-st} g^{(i)}(t) dt = x^i(s), \tag{8}$$

where $x(s)$ was defined in Theorem 1. Furthermore, the mean duration of the busy period initiated by one packet is given by

$$E(\mathbf{H}^{(1)}) = \frac{\rho}{\lambda(1 - \rho)}.$$

Before proceeding further, let us give a brief proof of the result in (8). Suppose that the busy period starts with only one packet. Then,

$$g^{(1)}(t) = \int_0^{\infty} \sum_{n=0}^{\infty} y_n(t) a(t) \left[\underbrace{g^{(1)}(t) * \cdots * g^{(1)}(t)}_{n} \right] dt,$$

where $a(t) = \sum_{j=0}^{\infty} a_j(t)$ is the pdf of the generalized service time. Therefore, its LST is given by,

$$g^{(1)}(s) = \int_0^{\infty} \sum_{n=0}^{\infty} e^{-(\lambda+s)t} \frac{\left(\lambda g^{(1)}(s) t\right)^n}{n!} a(t) dt = a^*(g^{(1)}(s), s),$$

and as a result, $g^{(1)}(s) = x(s)$ due to Theorem 1. It is easy to see that, if the busy period starts with $i\,(>1)$ customers, then it can be considered as the sum of i busy periods each of which starts with only one customer (see [41]).

We are now ready to give an expression for the pdf of the regeneration cycle, defined earlier. Let $\widetilde{w}(t)dt = P(t < \widetilde{W} \leq t + dt)$. Then,

$$\widetilde{w}(t) = \lambda e^{-\lambda t} (1 - I(t)) * g^{(1)}(t) + e^{-\lambda t} i(t) * \widetilde{c}(t), \tag{9}$$

where $\widetilde{c}(t)dt = P(t < \widetilde{C} \leq t + dt)$ and $\widetilde{C}$ is the time elapsed from the epoch a close-down period begins until the epoch the next inactivity period is about to begin. Let also $\widetilde{V}$ be the time elapsed from the epoch the server enters the short DRX period until the epoch the next inactivity period is about to begin, and $\widetilde{v}(t)dt = P(t < \widetilde{V} \leq t + dt)$. Then,

$$\widetilde{c}(t) = e^{-\lambda t} c(t) * \widetilde{v}(t) + \lambda e^{-\lambda t} (1 - C(t)) * \sum_{k=0}^{\infty} y_k(t) s(t) * g^{(k+1)}(t). \tag{10}$$

Furthermore,

$$\widetilde{v}(t) = \sum_{k=1}^{\infty} y_k(t) v_0(t) * \varphi_k(t) + e^{-\lambda t} v_0(t) * \sum_{k=1}^{\infty} y_k(t) v_0(t) * \varphi_k(t)$$

$$+ \cdots + \underbrace{e^{-\lambda t} v_0(t) * \cdots * e^{-\lambda t} v_0(t)}_{N-1} * \sum_{k=1}^{\infty} y_k(t) v_0(t) * \varphi_k(t)$$

$$+ \underbrace{e^{-\lambda t} v_0(t) * \cdots * e^{-\lambda t} v_0(t)}_{N} * \left[\sum_{k=1}^{\infty} y_k(t) v_1(t) * \varphi_k(t) + e^{-\lambda t} v_1(t) * \sum_{k=1}^{\infty} y_k(t) v_1(t) * \varphi_k(t) + \cdots \right] \tag{11}$$

where $\varphi_k(t) = \sum_{m=0}^{\infty} y_m(t) s(t) * g^{(k+m)}(t)$.

Then, the corresponding LSTs $\widetilde{w}^*(s) = \int_0^\infty e^{-st} \widetilde{w}(t)dt$, $\widetilde{c}^*(s) = \int_0^\infty e^{-st}\widetilde{c}(t)dt$ and $\widetilde{v}^*(s) = \int_0^\infty e^{-st}\widetilde{v}(t)dt$ are given by

$$w^*(s) = \lambda \frac{(1 - i^*(s + \lambda))x(s)}{s + \lambda} + i^*(s + \lambda)\widetilde{c}^*(s),$$

$$\widetilde{c}^*(s) = c^*(s + \lambda)\widetilde{v}^*(s) + \lambda x(s) \left(\frac{1 - c^*(s + \lambda)}{s + \lambda} \right) s^*(s + \lambda - \lambda x(s)), \tag{12}$$

$$\widetilde{v}^*(s) = \widehat{v}^*(x(s), s),$$

while the expected values of the above quantities are

$$E(\widetilde{W}) = \frac{1 - i^*(\lambda)}{\lambda(1 - \rho)} + i^*(\lambda)E(\widetilde{C}),$$

$$E(\widetilde{C}) = c^*(\lambda)E(\widetilde{V}) + \frac{(1 - c^*(\lambda))(1 + \lambda\overline{s})}{\lambda(1 - \rho)}, \tag{13}$$

$$E(\widetilde{V}) = \frac{\left[\overline{s} + \frac{(1 - (v_0^*(\lambda))^N)\overline{v}_0}{1 - v_0^*(\lambda)} + \frac{(v_0^*(\lambda))^N \overline{v}_1}{1 - v_1^*(\lambda)} \right]}{1 - \rho}.$$

5. The embedded Markov Chain

Let T_n be the epoch at which the nth packet is successfully delivered to the UE. These are the epochs at which a generalized service completion occurs and the server is ready for a "new service". Let X_n be the number of packets just after T_n. Then,

$$X_{n+1} = \begin{cases} X_n - 1 + A_{n+1}, & \text{if } X_n > 0, \\ A_{n+1}, & \text{if } X_n = 0, \ Q_n > 0, \\ \widehat{R}_{n+1} + S_{n+1} + A_{n+1} - 1, & \text{if } X_n = 0, \ Q_n = 0, \ C_n = 0, \\ S_{n+1} + A_{n+1}, & \text{if } X_n = 0, \ Q_n = 0, \ C_n > 0, \end{cases} \tag{14}$$

where A_n is the number of packets that arrive during the generalized service time of the nth packet, S_n is the number of packets that arrive during the start-up period for the transmission of the nth packet, Q_n is the number of packets that arrive during the inactivity period, which is initiated after the service time of the nth packet, C_n is the number of packets that arrive during the close-down period, which is enabled upon expiration of the inactivity period, and $\widehat{R}_n$ is the number of packets that arrive during the total DRX period.

Let the total DRX period be the elapsed time from the epoch the UE enters the first short DRX cycle until the epoch a start-up period is enabled for the first time. Therefore, in order for the total DRX period to be terminated, at least one packet must arrive during either a short DRX cycle or a long DRX cycle.

Let

$$w_k = \lim_{n \to \infty} P(W_n = k) = P(W = k), \quad k = 0, 1, \ldots,$$

be the limiting probability of W_n. Define also the probability mass functions

$$\begin{array}{ll} r_k = P(\widehat{R} = k), \quad k = 1, 2, \ldots, & a_k = P(A = k), \quad k = 0, 1, \ldots, \\ q_k = P(Q = k), \quad k = 0, 1, \ldots, & c_k = P(C = k), \quad k = 0, 1, \ldots, \\ s_k = P(S = k), \quad k = 0, 1, \ldots, & \pi_k = P(X = k), \quad k = 0, 1, \ldots, \end{array} \tag{15}$$

and the probability generating functions for $|z| \leq 1$,

$$\Pi(z) = \sum_{k=0}^\infty \pi_k z^k, \qquad A(z) = \sum_{k=0}^\infty a_k z^k,$$

$$\widehat{R}(z) = \sum_{k=1}^\infty r_k z^k, \qquad S(z) = \sum_{k=0}^\infty s_k z^k.$$

The state transition probabilities of X_n are given by

$$p_{jk} = P(X_{n+1} = k | X_n = j) = \begin{cases} a_{k-j+1}, & \text{if } j \geq 1, k \geq j - 1, \\ 0, & \text{if } j \geq 1, 0 \leq k \leq j - 1, \\ b_k, & \text{if } j = 0, k \geq 0 \end{cases} \tag{16}$$

where b_k, $k \geq 0$ is the probability that k customers arrived from the epoch the inactivity timer was enabled until the first packet departure epoch. Note that

$$b_k = (1 - q_0)a_k + q_0 \left[(1 - c_0) \sum_{v=0}^k s_v a_{k-v} + c_0 \sum_{v=1}^{k+1} \sum_{m=0}^{k+1-v} r_v s_m a_{k+1-m-v} \right]. \tag{17}$$

It is clear that $\{X_n, \, n = 0, 1, \ldots\}$ is an irreducible and aperiodic discrete time Markov chain (DTMC). Using (16), the balance equations are given by

$$\pi_k = \pi_0 b_k + \sum_{\nu=1}^{k+1} \pi_\nu a_{k-\nu+1}. \tag{18}$$

Forming the generating functions, we arrive at

$$(z - A(z)) \, \Pi(z) = \pi_0 A(z) \left[z(1 - q_0) + q_0 S(z) \left(z(1 - c_0) + c_0 \widehat{R}(z) \right) - 1 \right]. \tag{19}$$

Note that $A(z) = a^*(z, 0)$. Let us assume that $\rho < 1$. We will prove later that this condition is sufficient and necessary for the Markov chain to be ergodic. Since $\rho < 1$, using Theorem 1, the equation $z - A(z) = 0$ never vanishes inside the unit disk. Moreover, $q_0 = \int_0^\infty e^{-\lambda t} dI(t) = i^*(\lambda)$, $c_0 = \int_0^\infty e^{-\lambda t} dC(t) = c^*(\lambda)$. Thus,

$$\Pi(z) = \frac{\pi_0 A(z) \left[z \left(1 - i^*(\lambda) \right) + i^*(\lambda) S(z) \left(z(1 - c^*(\lambda)) + c^*(\lambda) \widehat{R}(z) \right) - 1 \right]}{z - A(z)}. \tag{20}$$

Since $s_k = \int_0^\infty y_k(t) dS(t)$, we conclude in $S(z) = s^*(\lambda - \lambda z)$. Define by $\widetilde{V}_0$ and $\widetilde{V}_1$ the number of packets that arrive during a short DRX and a long DRX cycle respectively, and denote by $v_m^{(k)} = P(\widetilde{V}_m = k)$, $m = 0, 1$, $k = 0, 1, \ldots$, their probability mass functions. It is easy to prove that their pgfs are given by $V_m(z) = v_m^*(\lambda - \lambda z)$, $m = 0, 1$.

Moreover, since the total DRX period will be terminated if and only if at least one packet arrives either during the short DRX period (which consists of at most N short DRX cycles) or during a long DRX cycle (in case the short DRX period expires without a packet arrival), we can obtain

$$r_k = v_0^{(k)} + v_0^{(0)} v_0^{(k)} + (v_0^{(0)})^2 v_0^{(k)} + \cdots + v_0^{(N-1)} v_0^{(k)}$$
$$+ (v_0^{(0)})^N \left[v_1^{(k)} + v_1^{(0)} v_1^{(k)} + (v_1^{(0)})^2 v_1^{(k)} + \cdots \right], \quad k = 1, 2, \ldots. \tag{21}$$

Eq. (21) is rearranged to yield

$$r_k = v_0^{(k)} \frac{1 - (v_0^{(0)})^N}{1 - v_0^{(0)}} + v_1^{(k)} \frac{(v_0^{(0)})^N}{1 - v_1^{(0)}}. \tag{22}$$

Since $v_m^{(0)} = \int_0^\infty y_0(t) dV_m(t) = v_m^*(\lambda)$, $m = 1, 2$, we arrive at,

$$\widehat{R}(z) = \frac{\left(1 - (v_0^*(\lambda))^N \right) \left(v_0^*(\lambda - \lambda z) - v_0^*(\lambda) \right)}{1 - v_0^*(\lambda)} + \frac{(v_0^*(\lambda))^N \left(v_1^*(\lambda - \lambda z) - v_1^*(\lambda) \right)}{1 - v_1^*(\lambda)}. \tag{23}$$

In order to prove positive recurrence, we use Foster's method [42], which states that an irreducible and aperiodic DTMC is positive recurrent if there exists a non-negative solution of the system

$$\sum_{k=0}^{\infty} p_{jk} x_k \le x_j - 1, \quad (j \neq 0) \tag{24}$$

such that $\sum_{k=0}^{\infty} p_{0k} x_k < \infty$.

Setting $x_k = \frac{k}{1-\rho}$, we arrive at

$$\begin{aligned}
\sum_{k=0}^{\infty} p_{jk} x_k &= \frac{j - 1 + \rho}{1 - \rho} = x_j - 1, \\
\sum_{k=0}^{\infty} p_{0k} x_k &= \sum_{k=0}^{\infty} \frac{k b_k}{1 - \rho} < \infty,
\end{aligned} \tag{25}$$

since $\sum_{k=0}^{\infty} k b_k = \rho + i^*(\lambda) \left[\lambda \bar{s} + c^*(\lambda)(E(\widehat{R}) - 1) \right]$. Hence, it follows that the DTMC is ergodic when $\rho < 1$. Since $\Pi(1) = 1$, using (20) we obtain,

$$\pi_0 = \frac{1 - \rho}{1 + i^*(\lambda) \left[\lambda \bar{s} + c^*(\lambda)(E(\widehat{R}) - 1) \right]}, \tag{26}$$

where,

$$E(\widehat{R}) = \frac{\lambda \bar{v}_0 \left(1 - (v_0^*(\lambda))^N \right)}{1 - v_0^*(\lambda)} + \frac{\lambda \bar{v}_1 (v_0^*(\lambda))^N}{1 - v_1^*(\lambda)}.$$

But $\pi_0 > 0$ when the DTMC is ergodic and therefore, $1 - \rho > 0$; thus, $\rho < 1$ is a necessary and a sufficient condition for ergodicity.

Note that the function

$$F(z) = z(1 - i^*(\lambda)) + i^*(\lambda)S(z)\left(z(1 - c^*(\lambda)) + c^*(\lambda)\widehat{R}(z)\right),$$ (27)

arising in the numerator of $\Pi(z)$, has an interesting probabilistic interpretation. More precisely, $F(z)$ is the pgf of the number of packets that arrive from the time instant the inactivity timer is enabled, until the time instant the eNB is ready to transmit the packets to the UE. Clearly, if F is the corresponding time period, $N(F)$ is the number of packets that arrive during F and $f_j(t)dt = P(t < F \leq t + dt, N(F) = j)$ then,

$$f_j(t) = \lambda e^{-\lambda t}(1 - I(t))\delta_{\{j=1\}} + e^{-\lambda t}i(t) * \lambda e^{-\lambda t}(1 - C(t)) * y_{j-1}(t)s(t) + e^{-\lambda t}i(t) * e^{-\lambda t}c(t) * \widehat{v}_j(t).$$ (28)

Thus, $F^*(z, s) = \sum_{j=1}^\infty \int_0^\infty e^{-st}f_j(t)z^j dt$ is given by

$$F^*(z, s) = \lambda z \frac{1 - i^*(s + \lambda)}{s + \lambda} + i^*(s + \lambda)\left[\lambda z \frac{1 - c^*(s + \lambda)}{s + \lambda}s^*(s + \lambda - \lambda z) + c^*(s + \lambda)\widehat{v}^*(z, s)\right].$$ (29)

Observe that $F(z) = F^*(z, 0)$. Note also that $\widehat{v}^*(z, 0) = s^*(\lambda - \lambda z)\widehat{R}(z) = S(z)\widehat{R}(z)$.

5.1. Waiting time

In the sequel, we derive the LST of the waiting time distribution of a tagged packet. Let D be the total waiting time of a tagged packet, and $D(t)$ its cdf. When the tagged packet is successfully delivered to the UE, the probability of leaving behind k packets is π_k. However, these k packets arrived during its waiting time D. Therefore, $\pi_k = \int_0^\infty y_k(t)dD(t)$. Thus,

$$\Pi(z) = d^*(\lambda - \lambda z) = d_Q^*(\lambda - \lambda z)A(z),$$ (30)

where $d_Q^*(\lambda - \lambda z)$ is the pgf of the number of customers that arrive during the waiting period of the tagged packet in the buffer. Then,

$$d^*(u) = \Pi(1 - u/\lambda) = \left\{\frac{u(1 - \rho)}{u - \lambda + \lambda\widetilde{A}(u)}\right\}\widetilde{A}(u)\frac{\lambda(1 - \widetilde{F}(u))}{u\left(1 + i^*(\lambda)\left[\lambda\bar{s} + c^*(\lambda)(E(\widehat{R}) - 1)\right]\right)},$$ (31)

where $\widetilde{A}(u) = A(1 - \frac{u}{\lambda}) = a^*(1, u)$ and $\widetilde{F}(u) = F(1 - u/\lambda)$.

5.2. A decomposition result

Clearly, our model is an unreliable queueing system with a modified multiple vacation scheme (DRX cycles), a timer (the inactivity period), start-up and close-down periods. It is well known that one of the fundamental results on vacation models is the stochastic decomposition property [43]. According to this property, the stationary queue length and the stationary waiting time can be decomposed in two or more independent random variables. One of them is the stationary queue length (waiting time) in the system without vacations.

Our model satisfies the stochastic decomposition property, and the results are summarized in the following lemmas.

Lemma 2. *If $\rho < 1$, the queue length X can be decomposed into two independent random variables such that,*

$$X = X_{gb} + X_c,$$ (32)

where X_{gb} is the queue length during a generalized service time of a packet, and X_c is the additional queue length due to the effect of the inactivity period, the DRX periods, the start-up and close-down periods (i.e. the non-serving period). The pgfs of the probability mass functions of the above random variables are given by,

$$\Pi_{gb}(z) = \frac{(1 - \rho)(1 - z)A(z)}{A(z) - z}, \qquad \Pi_c(z) = \frac{1 - F(z)}{(1 - z)\left(1 + i^*(\lambda)\left[\lambda\bar{s} + c^*(\lambda)(E(\widehat{R}) - 1)\right]\right)}.$$ (33)

Clearly,

$$\Pi(z) = \Pi_{gb}(z)\Pi_c(z).$$ (34)

Proof. We will focus on the derivation of $\Pi_c(z)$, since $\Pi_{gb}(z)$ is the pgf of the number of customers for the unreliable $M/G/1$ model without multiple vacations, the timer, start-up and close-down periods. Borst and Boxma [33] showed that

$$\Pi_c(z) = \frac{G_0(z) - G_1(z)}{(1 - z)(E(Y_1) - E(Y_0))},$$

where Y_0 (Y_1) is the number of customers at the start (end) of a non-serving period, and $G_0(z)$ ($G_1(z)$) its pgf. In our case, $Y_0 = 0$, whereas (see (14) and (15))

$$Y_1 = \begin{cases} 1, & \text{with probability } 1 - i^*(\lambda) \\ \widehat{R} + S, & \text{with probability } i^*(\lambda)c^*(\lambda) \\ S + 1, & \text{with probability } i^*(\lambda)(1 - c^*(\lambda)). \end{cases}$$

Therefore, $G_0(z) = 1$, and after manipulations, $G_1(z) = F(z)$. Moreover, $E(Y_0) = 0$ and

$$E(Y_1) = 1 + i^*(\lambda)\left[\lambda\bar{s} + c^*(\lambda)(E(\widehat{R}) - 1)\right].$$

Combining these results we obtain

$$\Pi_c(z) = \frac{1 - F(z)}{(1 - z)\left(1 + i^*(\lambda)\left[\lambda\bar{s} + c^*(\lambda)(E(\widehat{R}) - 1)\right]\right)}. \quad \square$$

Similarly,

Lemma 3. *If $\rho < 1$, the waiting time D can be decomposed into two independent random variables such that*

$$D = D_{gb} + D_c, \tag{35}$$

where D_{gb} is the waiting time for the model without an inactivity timer, the DRX periods, the start-up and close-down periods. D_c is the additional delay due to the effect of the non-serving period. The LSTs of the cdfs of the above random variables are given by

$$d_{gb}^*(u) = \frac{u(1 - \rho)\widetilde{A}(u)}{u - \lambda + \lambda\widetilde{A}(u)}, \qquad d_c^*(u) = \Pi_c(1 - u/\lambda). \tag{36}$$

Clearly,

$$d^*(u) = d_{gb}^*(u)d_c^*(u). \tag{37}$$

6. Performance measures

In this section, we obtain some important measures that quantify the performance of the system. The UE's receiver activities are characterized by a regenerative process (see Ross [44]), which consists of the inactivity period, the close-down period, the start-up period, the short and the long DRX periods and the busy period. In order to calculate the long-run probabilities of the receiver's state, we are going to evaluate the mean residence time of the UE's receiver, at each of these states during a regeneration cycle.

Let $\widehat{I}$ be the residence time of UE's receiver in an inactive state (the inactivity period) during a regeneration cycle. Therefore, if $\widehat{i}(t)dt = P(t < \widehat{I} \leq t + dt)$,

$$\widehat{i}(t) = e^{-\lambda t}i(t) + \lambda e^{-\lambda t}(1 - I(t)). \tag{38}$$

The LST of (38) is given by $\widehat{i}^*(s) = \lambda\frac{1 - i^*(s+\lambda)}{s+\lambda} + i^*(s + \lambda)$. Then, $E(\widehat{I}) = \frac{1 - i^*(\lambda)}{\lambda}$.

Define by $\widehat{C}$ the elapsed time from the epoch the close-down period is enabled, until the epoch it is terminated during a regeneration cycle. If $\widehat{c}(t)dt = P(t < \widehat{C} \leq t + dt)$, then,

$$\widehat{c}(t) = e^{-\lambda t}c(t) + \lambda e^{-\lambda t}(1 - C(t)). \tag{39}$$

Thus, given that the inactivity timer has expired (with probability $i^*(\lambda)$), the mean residence time, say $E(\widehat{CL})$, of the UE's receiver in the close-down state during a regeneration cycle is given by,

$$E(\widehat{CL}) = i^*(\lambda)E(\widehat{C}) = i^*(\lambda)\frac{1 - c^*(\lambda)}{\lambda}. \tag{40}$$

Similarly, denote by $\widehat{V}_0$ the time period spent by the UE in the short DRX period during the regeneration cycle. Given that the inactivity period and the close-down period have expired (with probability $i^*(\lambda)c^*(\lambda)$), the pdf $\widehat{v}_0(t)dt = P(t < \widehat{V}_0 \leq t + dt)$ is given by,

$$\widehat{v}_0(t) = \underbrace{e^{-\lambda t}v_0(t) * \cdots * e^{-\lambda t}v_0(t)}_{N} + \sum_{k=1}^{\infty}y_k(t)v_0(t) + e^{-\lambda t}v_0(t) * \sum_{k=1}^{\infty}y_k(t)v_0(t)$$

$$+ e^{-\lambda t}v_0(t) * e^{-\lambda t}v_0(t) * \sum_{k=1}^{\infty}y_k(t)v_0(t) + \cdots + \underbrace{e^{-\lambda t}v_0(t) * \cdots * e^{-\lambda t}v_0(t)}_{N-1} * \sum_{k=1}^{\infty}y_k(t)v_0(t). \tag{41}$$

Then, the corresponding LST is given by,

$$\widehat{v}_0^*(s) = (v_0^*(s + \lambda))^N + \frac{\left(1 - (v_0^*(s + \lambda))^N\right)\left(v_0^*(s) - v_0^*(s + \lambda)\right)}{1 - v_0^*(s + \lambda)}. \tag{42}$$

Thus, the mean residence time, say $E(\widehat{SD})$, of the UE's receiver in the short DRX period during a regeneration cycle is given by,

$$E(\widehat{SD}) = i^*(\lambda)c^*(\lambda)E(\widehat{V}_0) = \frac{i^*(\lambda)c^*(\lambda)\left(1 - (v_0^*(\lambda))^N\right)\overline{v}_0}{1 - v_0^*(\lambda)}. \tag{43}$$

It is known that at the end of the sleep period of each short DRX cycle, the UE wakes up for a short period τ (this is the listening period or equivalently the on-period) to listen to the paging information from the network. Assume also that the UE's receiver spent K short DRX cycles in the short DRX period, where $1 \leq K \leq N$. Therefore, the mean residence time of the UE's receiver in the listening period during a regeneration cycle, given that UE is in the short DRX period, equals $E(K)\tau$. Moreover,

$$K = \begin{cases} 1, & \text{with probability } 1 - v_0^*(\lambda), \\ 2, & \text{with probability } (1 - v_0^*(\lambda))v_0^*(\lambda), \\ 3, & \text{with probability } (1 - v_0^*(\lambda))(v_0^*(\lambda))^2, \\ \vdots \\ N - 1, & \text{with probability } (1 - v_0^*(\lambda))(v_0^*(\lambda))^{N-2}, \\ N, & \text{with probability } (1 - v_0^*(\lambda))(v_0^*(\lambda))^{N-1} + (v_0^*(\lambda))^N. \end{cases}$$

Provided that the inactivity period and the close-down period have expired (with probability $i^*(\lambda)c^*(\lambda)$),

$$E(K) = \frac{i^*(\lambda)c^*(\lambda)\left(1 - (v_0^*(\lambda))^N\right)}{1 - v_0^*(\lambda)}. \tag{44}$$

Let $\widehat{V}_1$ be the time period spent by the UE in the long DRX period during a regeneration cycle. Note that in order for the long DRX period to be enabled, the short DRX period has to be expired first (with probability $(v_0^*(\lambda))^N$). If $\widehat{v}_1(t)dt = P(t < \widehat{V}_1 \leq t + dt)$ then,

$$\widehat{v}_1(t) = e^{-\lambda t}v_1(t) * \widehat{v}_1(t) + \sum_{k=1}^{\infty} y_k(t)v_1(t), \tag{45}$$

and its LST is $\widehat{v}_1^*(s) = \frac{v_1^*(s) - v_1^*(s + \lambda)}{1 - v_1^*(s + \lambda)}$. Thus, the mean residence time, say $E(\widehat{LD})$, of the UE's receiver in the long DRX period during a regeneration cycle is given by,

$$E(\widehat{LD}) = i^*(\lambda)c^*(\lambda)(v_0^*(\lambda))^N E(\widehat{V}_1) = \frac{i^*(\lambda)c^*(\lambda)(v_0^*(\lambda))^N \overline{v}_1}{1 - v_1^*(\lambda)}. \tag{46}$$

Assume that the UE spent M long DRX cycles during a regeneration cycle. Using Wald's theorem [44],

$$E(\widehat{LD}) = E(M)\overline{v}_1 \Rightarrow E(M) = \frac{i^*(\lambda)c^*(\lambda)(v_0^*(\lambda))^N}{1 - v_1^*(\lambda)}. \tag{47}$$

In the sequel we derive the mean residence time, say $E(\widehat{ST})$, of the UE's receiver in the start-up state during a regeneration cycle. If a packet arrives during the close-down period, a start-up period is enabled. In such a case, the mean residence time of the UE's receiver in the start-up state equals $i^*(\lambda)(1 - c^*(\lambda))\overline{s}$. Consider the case where the inactivity period and the close-down period have expired (with probability $i^*(\lambda)c^*(\lambda)$). In such a case, the UE enters the DRX mode. Then, in order for a start-up period to be initiated, a packet has to arrive either during the short DRX period (with probability $1 - (v_0^*(\lambda))^N$), or during the long DRX period (with probability $(v_0^*(\lambda))^N$). Thus,

$$E(\widehat{ST}) = i^*(\lambda)\overline{s}\left\{1 - c^*(\lambda) + c^*(\lambda)\left[1 - (v_0^*(\lambda))^N + (v_0^*(\lambda))^N\right]\right\} = i^*(\lambda)\overline{s}. \tag{48}$$

Finally, we are going to derive the mean residence time, say $E(\widehat{B})$, during which the UE's receiver is in the busy state during a regeneration cycle. Clearly, if a packet arrives during the inactivity period (with probability $1 - i^*(\lambda)$), a busy period with one customer is initiated. In such a case, the mean residence time of the UE's receiver in the busy state during a regeneration cycle equals $E(\widehat{B}_0) = (1 - i^*(\lambda))E(\mathbf{H}^{(1)}) = \frac{(1 - i^*(\lambda))\rho}{\lambda(1 - \rho)}$.

Consider the case that the inactivity period has expired, and the UE enters the close-down period. If a packet arrives during that period (with probability $i^*(\lambda)(1 - c^*(\lambda))$), the close-down period is stopped and a start-up period is initiated.

Then, the mean residence time in the busy state during a regeneration cycle is the sum of the generalized service times of the packets that arrive during the start-up period plus the generalized service time of the packet that terminates the close-down period. Thus, $E(\widehat{B}_1) = i^*(\lambda)(1 - c^*(\lambda))(1 + \lambda \bar{s}) E(\mathbf{H}^{(1)}) = \frac{i^*(\lambda)(1 - c^*(\lambda))(1 + \lambda \bar{s})\rho}{\lambda(1-\rho)}$.

Assume that the close-down period has now expired, and the UE enters the total DRX period (with probability $i^*(\lambda)c^*(\lambda)$). In such a case, in order for the UE to enter the busy state, at least one packet must arrive during the total DRX period. Then, the UE's receiver will remain in the busy state for a time period that is the sum of the generalized service times of the packets that arrive during the total DRX period, plus the sum of the generalized service times of the packets that arrive during the start-up period. Using Wald's theorem [44],

$$E(\widehat{B}_2) = i^*(\lambda)c^*(\lambda)\left(\lambda \bar{s} + E(\widehat{R})\right) E(\mathbf{H}^{(1)}). \tag{49}$$

Therefore, the total mean residence time of the UE's receiver in the busy state during a regeneration cycle is given by,

$$E(\widehat{B}) = E(\widehat{B}_0) + E(\widehat{B}_1) + E(\widehat{B}_2) = \rho E(\widetilde{W}). \tag{50}$$

6.1. Probabilities of the UE's receiver activities

Using the above results, we can obtain the probabilities of the UE's receiver activities in steady state. The UE's receiver can be in the following states: in the inactive state (IN), in the close-down state (CL), in the start-up state (ST), in the sleep state during a short DRX cycle (SD), in the sleep state during a long DRX cycle (LD), in the listening state (L) and in the busy state (B). Following the lines in Ross [44] (Theorem 3.7.1),

$$P_j = \lim_{t \to \infty} P(\text{the UE's receiver is in state } j \text{ at time } t)$$

$$= \frac{E(\text{amount of time in state } j \text{ during } \widetilde{W})}{E(\widetilde{W})}.$$

Then,

$$P_B = \frac{E(\widehat{B})}{E(\widetilde{W})} = \rho, \qquad P_{IN} = \frac{E(\widehat{I})}{E(\widetilde{W})},$$

$$P_{ST} = \frac{E(\widehat{ST})}{E(\widetilde{W})}, \qquad P_L = \frac{(E(M) + E(K))\tau}{E(\widetilde{W})}, \tag{51}$$

$$P_{SD} = \frac{E(\widehat{SD}) - E(K)\tau}{E(\widetilde{W})},$$

$$P_{LD} = \frac{E(\widehat{LD}) - E(M)\tau}{E(\widetilde{W})}, \tag{52}$$

$$P_{CL} = \frac{E(\widehat{CL})}{E(\widetilde{W})}.$$

It is easy to see that $\rho = \rho_1 + \rho_2$, where

$$\rho_1 = \frac{\lambda(1 - \beta^*(\theta))}{\theta(\beta^*(\theta) + p(1 - \beta^*(\theta)))}, \qquad \rho_2 = \frac{\lambda(1 - \beta^*(\theta))\bar{r}}{\beta^*(\theta) + p(1 - \beta^*(\theta))}. \tag{53}$$

Using similar arguments to those used in this section, we can prove that ρ_1 is the steady state probability that the UE's receiver is working properly (receiving a packet), whereas ρ_2 is the steady state probability that the UE's receiver is under recovery.

6.2. Power saving factor

We now compute a very important metric, called the power saving factor (PSF), which provides the percentage of time during which the UE is turned off, and as a consequence, does not consume power. Following Ross [44],

$$PSF = P_{SD} + P_{LD} = \frac{i^*(\lambda)c^*(\lambda)}{E(\widetilde{W})} \left[\frac{\left(1 - (v_0^*(\lambda))^N\right)(\bar{v}_0 - \tau)}{1 - v_0^*(\lambda)} + \frac{(v_0^*(\lambda))^N(\bar{v}_1 - \tau)}{1 - v_1^*(\lambda)} \right]. \tag{54}$$

6.3. Mean value analysis

In the sequel, we use the mean value analysis in order to calculate the expected waiting time of a packet in the system. For further reading about this method, see [45].

Let $\underline{D}(D)$ be the waiting time of an arriving packet in the queue (system). A newly arriving packet has first to wait for the residual service time of the packet in service (if any); then it continues to wait for the servicing of all packets which were already waiting in the queue on arrival. It is known by the PASTA property (Poisson Arrivals See Time Averages, see [46]) that with probability ρ the server is in the busy state on arrival. Let the random variable Z_U be the residual time (see [45]) of the random variable U, and $\underline{X}$ be the number of packets waiting in the queue at the arrival instant of the tagged packet.

If the tagged packet arrives during a short or a long DRX cycle, it has to wait for the server to finish the corresponding residual DRX cycle (either a short or a long) plus a start-up period, whereas if it arrives during a start-up period, it has to wait for the residual start-up period to be finished. Finally, in case the tagged packet arrives during the close-down period, it has to wait for a start-up period. Thus,

$$E(\underline{D}) = E(\underline{X})E(A) + \rho E(Z_A) + P_{CL}\bar{s} + P_{SD}(E(Z_{V_0}) + \bar{s}) + P_{LD}(E(Z_{V_1}) + \bar{s}) + P_{ST}E(Z_S), \tag{55}$$

where $E(Z_U) = \frac{E(U^2)}{2E(U)}$, for any random variable U. Using Little's law and Eqs. (51), (52), Eq. (55) becomes,

$$E(\underline{D}) = \frac{\rho^{(2)}}{2\lambda(1-\rho)} + \frac{i^*(\lambda)\left[\lambda\bar{s}^{(2)} + 2\bar{s}\left(1 + c^*(\lambda)\left(E(\widehat{R}) - 1\right)\right) + c^*(\lambda)\frac{\partial^2\widehat{R}(z)}{\partial z^2}|_{z=1}/\lambda\right]}{2\left(1 + i^*(\lambda)\left[\lambda\bar{s} + c^*(\lambda)\left(E(\widehat{R}) - 1\right)\right]\right)}, \tag{56}$$

where,

$$\rho^{(2)} = \frac{\lambda^2\left[2\left(\dot{\beta}^*(\theta) + \frac{1-\beta^*(\theta)}{\theta}\right)(1 + \theta\bar{r}) + (1 - \beta^*(\theta))\bar{r}^{(2)}\theta\right]}{\theta\left(\beta^*(\theta) + p(1 - \beta^*(\theta))\right)}$$

$$+ \frac{2(1-p)\lambda^2(1 - \beta^*(\theta))(1 + \theta\bar{r})\left(\dot{\beta}^*(\theta) + \frac{1-\beta^*(\theta)}{\theta}\right)}{\theta\left(\beta^*(\theta) + p(1 - \beta^*(\theta))\right)^2},$$

$$\frac{\partial^2\widehat{R}(z)}{\partial z^2}\bigg|_{z=1} = \frac{\left(1 - (v_0^*(\lambda))^N\right)\lambda^2\bar{v}_0^{(2)}}{1 - v_0^*(\lambda)} + \frac{(v_0^*(\lambda))^N\lambda^2\bar{v}_1^{(2)}}{1 - v_1^*(\lambda)},$$

and $\dot{\beta}^*(\theta) = \frac{d}{ds}\beta^*(s + \theta)|_{s=0}$.

Clearly $E(D) = E(\widetilde{A}) + E(\underline{D})$. Moreover, using Little's law, we obtain the expected number of packets in the system $E(X) = \rho + \lambda E(\underline{D})$.

6.4. Average power consumption

Let C_I, C_{CL}, C_{ST}, C_{SD}, C_{LD}, C_L and C_B the power consumption (in mW) of the UE's receiver during the inactivity period, the close-down period, the start-up period, the sleep period in a short DRX cycle, the sleep period in a long DRX cycle, the listening period (this is the on-period at the end of each DRX cycle), and in the busy period, respectively. Then, the average power consumption (APC) of the UE's receiver is given by,

$$APC = P_{IN}C_I + P_{CL}C_{CL} + P_{ST}C_{ST} + P_{SD}C_{SD} + P_{LD}C_{LD} + \rho C_B + P_L C_L. \tag{57}$$

We are now going to show that the DRX in LTE will achieve greater power savings than the scheme used in UMTS systems, at the cost of a prolonged delay to wake up. The key difference, regarding the DRX operation, is that in UMTS systems, there is only a single DRX period.

Let $APC^{(*)}$ and $E^{(*)}(D)$ be the average power consumption and the expected waiting time per packet respectively for the model with a single DRX period. From here on, we assume that the duration of a DRX cycle in the DRX in UMTS scheme, equals the duration of the short DRX cycle V_0, used in the DRX in the LTE scheme. Then,

$$APC^{(*)} = \frac{E^{(*)}(\widehat{I})}{E^{(*)}(\widetilde{W})}C_I + \frac{E^{(*)}(\widehat{CL})}{E^{(*)}(\widetilde{W})}C_{CL} + \frac{E^{(*)}(\widehat{ST})}{E^{(*)}(\widetilde{W})}C_{ST} + \frac{E^{(*)}(DRX) - E^{(*)}(K)\tau}{E^{(*)}(\widetilde{W})}C_{SD} + \frac{E^{(*)}(\widehat{B})}{E^{(*)}(\widetilde{W})}C_B + \frac{E^{(*)}(K)\tau}{E^{(*)}(\widetilde{W})}C_L, \tag{58}$$

where $E^{(*)}(\widehat{I}) = E(\widehat{I}), E^{(*)}(\widehat{CL}) = E(\widehat{CL}), E^{(*)}(\widehat{ST}) = E(\widehat{ST})$ and

$$E^{(*)}(DRX) = \frac{i^*(\lambda)c^*(\lambda)\bar{v}_0}{1 - v_0^*(\lambda)}, \qquad E^{(*)}(K) = \frac{E^{(*)}(DRX)}{\bar{v}_0}, \tag{59}$$

$$E^{(*)}(\widehat{B}) = \frac{\rho\left[(1 - i^*(\lambda) + i^*(\lambda)(1 - c^*(\lambda)))\left(1 - v_0^*(\lambda)\right) + \lambda i^*(\lambda)\left(c^*(\lambda)\bar{v}_0 + \bar{s}(1 - v_0^*(\lambda))\right)\right]}{\lambda(1-\rho)\left(1 - v_0^*(\lambda)\right)}, \tag{60}$$

$$E^{(*)}(\widetilde{W}) = E^{(*)}(\widehat{I}) + E^{(*)}(\widehat{CL}) + E^{(*)}(\widehat{ST}) + E^{(*)}(DRX) + E^{(*)}(\widehat{B}).$$

Table 1
Comparison of the analytical and simulation results.

λ	0.1	0.15	0.2	0.25	0.3	0.35
PSF (Analytical)	0.8765	0.8343	0.7982	0.7666	0.7386	0.7136
PSF (Simulation)	0.877	0.8339	0.7985	0.7673	0.7391	0.7131
Error	0.057%	0.048%	0.0376%	0.0912%	0.0677%	0.0701%

The gain in energy, and the cost of the delay by the employment of DRX in LTE is given by,

$$\widehat{G} = \frac{APC^{(*)} - APC}{APC^{(*)}}, \qquad \widehat{D} = \frac{E(D) - E^{(*)}(D)}{E(D)},$$

where

$$E^{(*)}(D) = \rho + \frac{\rho^{(2)}}{2(1-\rho)} + \frac{i^*(\lambda)\left[\lambda^2\bar{s}^{(2)} + 2\lambda\bar{s}\left(1 + c^*(\lambda)\left(\frac{\lambda\bar{v}_0}{1-v_0^*(\lambda)} - 1\right)\right) + \frac{c^*(\lambda)\lambda^2\bar{v}_0^{(2)}}{1-v_0^*(\lambda)}\right]}{2\left(1 + i^*(\lambda)\left[\lambda\bar{s} + c^*(\lambda)\left(\frac{\lambda\bar{v}_0}{1-v_0^*(\lambda)} - 1\right)\right]\right)}.$$

6.5. Energy efficiency and the cost function

In order to provide a metric that gives a fair trade off between energy saving and delay, we consider the following metric, called the energy efficiency. A similar metric was introduced in [47] for measuring the efficiency of the sleep mode operation in IEEE 802.16e. Let η_c be the energy efficiency, defined by

$$\eta_c = \widehat{G} - c\widehat{D}, \tag{61}$$

where c is a delay penalty, which reflects to what extent low packet delay is preferred over energy saving.

For delay sensitive applications, this penalty must be set relatively high. If the primary focus is to extend the battery lifetime, then c should be small. Thus, when $\eta_c > 0$, the DRX in the LTE scheme is efficient to be used.

Let us denote by t_0, t_1, the holding cost of each packet present in the system, and the cost per unit power that the UE consumes respectively. The following function gives the total cost of the system,

$$TC = t_0 E(X) + t_1 APC. \tag{62}$$

6.6. System's availability

An important measure of system's reliability is the percentage of time the server is available for requests. The steady state availability of the system is the steady state probability, say AV, that the UE's receiver is either working properly, or in the listening state, or in the inactive state or in the start-up state or in the close-down state. Thus,

$$AV = \rho_1 + P_L + P_{IN} + P_{ST} + P_{CL}. \tag{63}$$

7. Numerical results

We now provide numerical results in order to evaluate the performance of the system. We focus on energy metrics, which are the main concepts of this work.

The analytical model has been validated against simulations. These simulation experiments are based on a discrete-event simulation model (including the following events: the packet arrival, the packet departure, the failure, the recovery, the start-up, the close-down, the sleep in a short and in a long DRX period, the listening, and the wake up), which simulates the UE power saving behavior, according to the DRX in LTE mechanism. The output measures of the simulation are the length T_0 of the total observation period in the simulation run, and the length T_s of the total sleep period within T_0. These output measures are used to compute the power saving factor $PSF = \frac{T_s}{T_0}$. This simulation model is similar to the one developed in [48], and further details are omitted.

In what follows, the service time, the recovery time, the short and the long DRX cycle, the start-up time and the close-down time are assumed to be exponentially distributed random variables. Furthermore, we set the following default parameters: $\bar{b} = \bar{c} = 0.2$ s, $\bar{r} = 0.05$ s, $p = 0.2$, $\theta = 0.1$, $\tau = 0.02$ s, $\bar{s} = 0.2$ s, $C_B = 300$ mW, $C_{CL} = C_{SD} = 30$ mW, $C_{LD} = 15$ mW, $C_{ST} = 45$ mW and $C_L = 105$ mW.

Table 1 compares the analytical and simulation results. The table indicates a very good match between the analytical and simulation results.

The investigation of the DRX in LTE performance is based on the analytical model. Fig. 6 shows the way the PSF varies for different values of λ. As expected, when λ increases, the PSF decreases, a fact with a negative contribution to the energy efficiency of the UE. Specifically, when the mean inactivity period ($\bar{i}$) increases, the PSF decreases faster. Thus, the larger the value of the $\bar{i}$, the more the UE remains awake and available for possible requests. In such a case, the UE will consume more power.
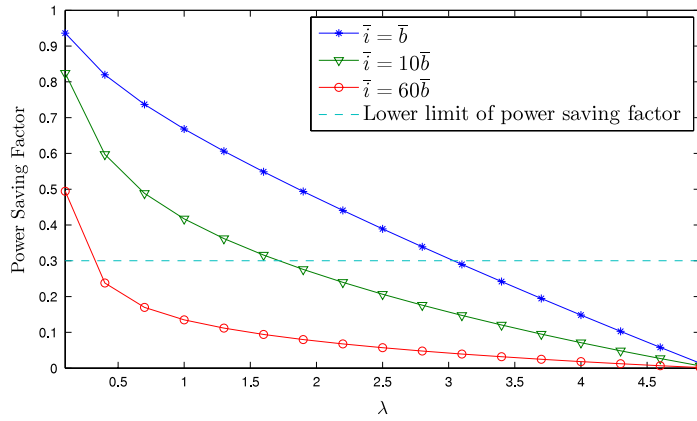
Fig. 6. PSF vs. λ for $N = 4$, $\overline{v}_0 = 2$ s, $\overline{v}_1 = 4$ s.
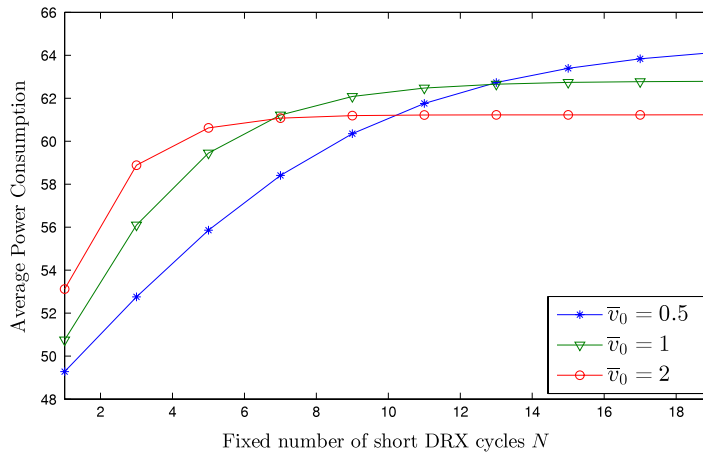

Fig. 7. APC vs. N for $\lambda = 0.5$, $\overline{i} = 0.2$ s, $\overline{v}_1 = 4$ s.

The dashed horizontal line corresponds to an arbitrary preset threshold, and splits the operation zone into a DRX inefficient, below the line, and a DRX efficient zone, above the line. Clearly, when the $\overline{i}$ takes small values, the DRX in the LTE scheme is efficient to be used for a larger range of values of λ.

Fig. 7 shows the way the APC is affected by the fixed number of short DRX cycles N for different values of $\overline{v}_0$. Note that by increasing the fixed number N, the APC is increasing. The reason is that in such a case, the UE does not switch fast in the long DRX period, which is the dominant source of power conservation for our model. However, we can observe that when N surpasses an upper level, the APC seems to be stabilized.

Fig. 8 displays the effect of N on the energy gain by using the DRX in the LTE scheme, instead of using the DRX in the UMTS scheme. Observe that as the number of short DRX cycles increases, the energy gain decreases. This decrease in the energy gain becomes more apparent, especially for large values of $\overline{v}_0$.

In Figs. 9 and 10 the efficiency $\eta_{0.05}$ is shown as a function of the $\overline{v}_0$, and as a function of the fixed number of short DRX cycles N, respectively. According to these figures, it is efficient to use the DRX in the LTE scheme, instead of DRX in UMTS when $\eta_{0.05} > 0$.

8. Conclusion

In this paper, we studied the performance of the Discontinuous Reception (DRX) mechanism in 3GPP LTE wireless systems, in the presence of failures due to transient device faults and due to wireless link errors. An unreliable $M/G/1$ queueing model with multiple vacations, a timer, start-up and close-down periods was used in order to study the effects of the DRX parameters and the failures, that may cause the retransmission of a packet, on output measures including the average power consumption, the energy efficiency and the power saving factor. The analytical approach was validated against simulations, and several numerical examples were presented. Our study indicated that with proper parameter settings, the DRX scheme can effectively reduce the power consumption of the UE.
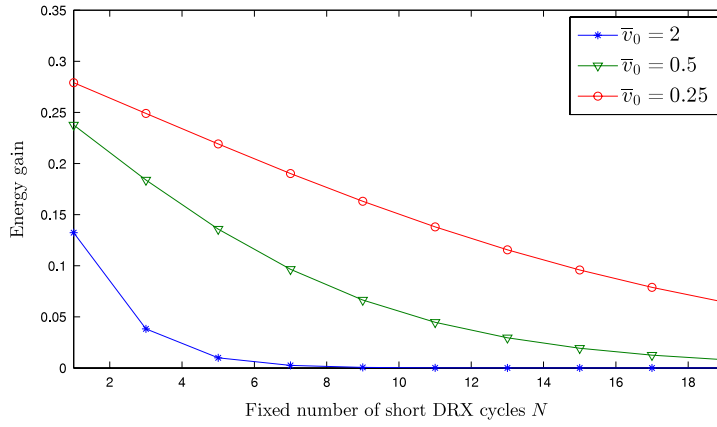
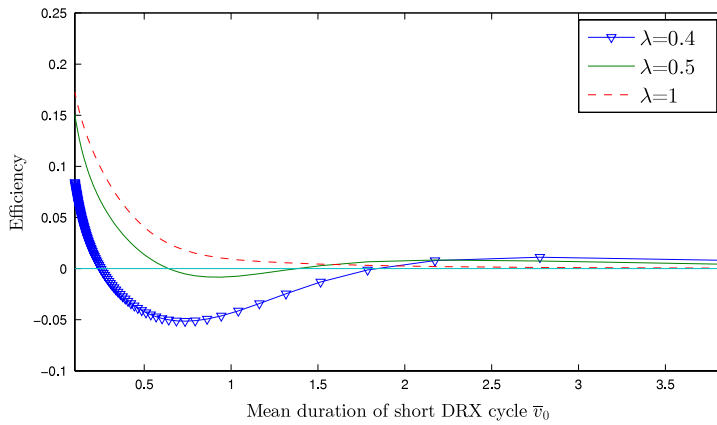**Fig. 8.** Energy gain vs. N for $\lambda = 0.5$, $\overline{v}_1 = 4$ s, $\overline{i} = 0.2$ s.

Fig. 9. Efficiency $\eta_{0.05}$ vs. $\overline{v}_0$ for $N = 4$, $\overline{v}_1 = 4$ s, $\overline{i} = 0.2$ s.
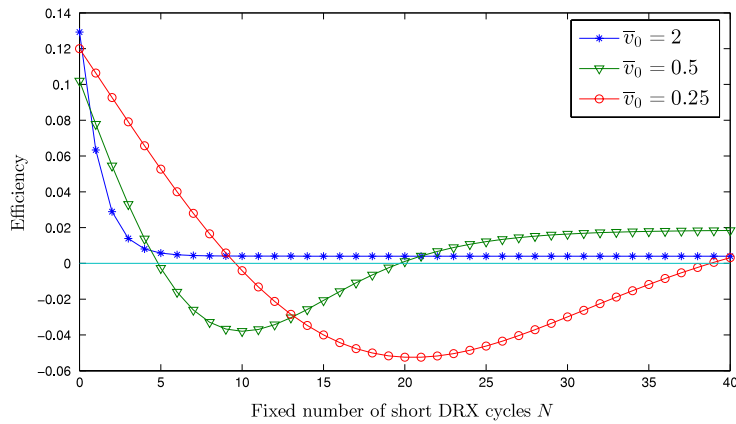
Fig. 10. Efficiency $\eta_{0.05}$ vs. N for $\lambda = 0.5$, $\overline{v}_1 = 4$ s, $\overline{i} = 0.2$ s.

Acknowledgments

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